Fall 2024

For students **entering** Calculus

Calculus Summer Review

Topics For Review

- Topic 1: Exponent Rules & Simplification
- Topic 2: Factoring GCF & Quadratics
- Topic 3: Special Factoring Cases
- Topic 4: Solving Equations for Roots/Zeros
- Topic 5: Parent Functions
- Topic 6: Function Transformations
- Topic 7: Composition of Functions
- Topic 8: Unit Circle
- Topic 9: Inverse Trig
- Topic 10: Solving Trigonometric Equations

Topic 1: Exponent Rules & Simplification

All final answers should have positive exponents. If your answer contains fractional exponents, you should be able to write the exponent in radical form as well.

1. $3yx^2 \cdot y^{-4}$

2. $3x^2y^2 \cdot x^2y^3$

3. $3m^{-3}n^2 \cdot 4m^3n^4$

4. $\left(2x^2\right)^3 \cdot x^{-1}$

5. $2yy^{-1}$

6. $\left(a^2b^2\right)^{-1} \cdot a$

7. $\dfrac{3a^{3/2}b^{3/2} \cdot 4a^{5/4}b^{3/4}}{3a^{-1}b^{-1/3}}$

8. $\dfrac{2xy^{2/3}}{x^{-1/2}y^2 \cdot 2xy}$

9. $\dfrac{4b^{-2}}{\left(2a^{-1}b^4\right)^4}$

10. $\dfrac{x^{-2}y^{-2}}{3y^{-2} \cdot 3x^{-4}}$

11. $\dfrac{3a^4b^3}{4ab^2 \cdot 3b^2 \cdot 4ba^3}$

12. $\dfrac{3u^{5/3}v^{4/5}}{4v^{-1/5} \cdot 6u^{7/3}}$

Topic 2: Factoring GCF & Quadratics

1. $x^2+15x+56$

2. $x^2-3x-10$

3. x^2+x-72

4. $x^2-11x+30$

5. x^2+x-6

6. $x^2-13x+36$

7. $x^2+15x-100$

8. $x^2-2x-48$

9. $-x^2-4x+45$

10. $-x^3+2x^2+80x$

11. $4x^4+4x^3-120x^2$

12. $4x^3-32x^2+64x$

13. $2x^2-3x-2$

14. $3x^2-11x+6$

15. $5x^2+8x+3$

16. $4x^2+x-5$

17. $10x^2+19x+6$

18. $6x^2+17x+12$

19. $5x^2-8x-4$

20. $4x^2+12x+5$

Topic 3: Special Factoring Cases

1. $x^3 + 8$

2. $x^3 - 64$

3. $27x^3 - 125y^3$

4. $x^4 - 11x^2 - 80$

5. $x^3 + 3x^2 - 9x - 27$

6. $35ab - 56a - 5b + 8$

7. $6x^4 + 13x^2 + 5$

8. $x^4 - 13x^2 + 36$

9. $121x^2 - 400$

10. $225 - 16x^2$

Topic 4: Solving Equations for Roots/Zeros

1. $7x^2 - 3x = 0$

2. $x^2 + 8x - 20 = 0$

3. $3x^2 - x - 14 = 0$

4. $4x(x-2) - 5x(x-1) = 2$

5. $x^2 + 6x + 4 = 0$

6. $2x^2 - (x+2)(x-3) = 12$

7. $x + \frac{1}{x} = \frac{13}{6}$

8. $\frac{1}{x^2} - \frac{1}{x} = 6$

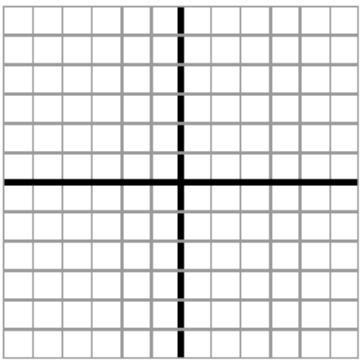
9. $x^4 - 9x^2 + 8 = 0$

10. $x - 10\sqrt{x} + 9 = 0$

Topic 5: Parent Functions

Sketch a graph and fill out the function, domain, and range for each parent function given below.

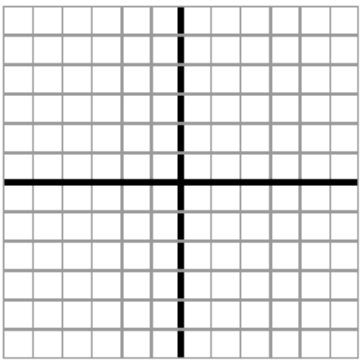
1. Constant $y = number$



Domain:

Range:

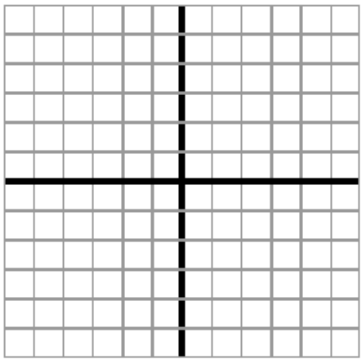
2. Linear $y = x$



Domain:

Range:

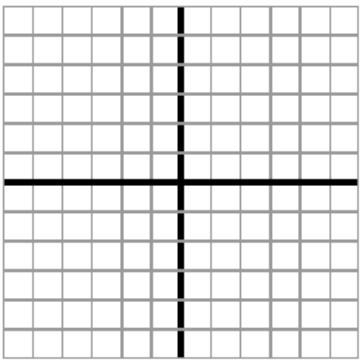
3. Quadratic $y = x^2$



Domain:

Range:

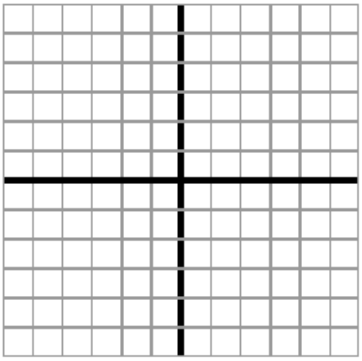
4. Square Root $y = \sqrt{x}$



Domain:

Range:

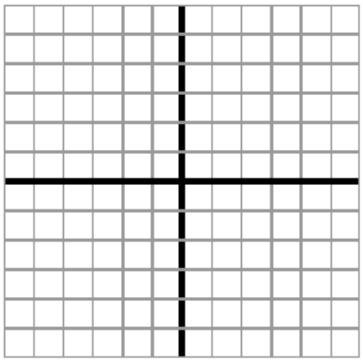
5. Cubic $y = x^3$



Domain:

Range:

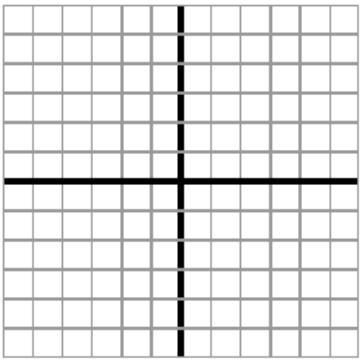
6. Cubic Root $y = \sqrt[3]{x}$



Domain:

Range:

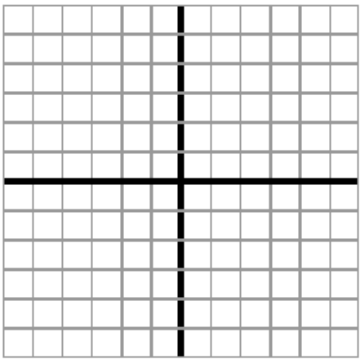
7. Absolute Value $y = |x|$



Domain:

Range:

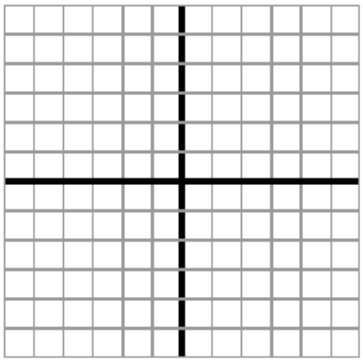
8. Exponential $y = b^x$



Domain:

Range:

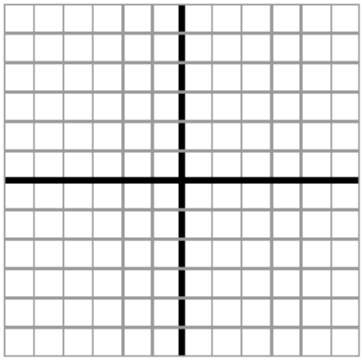
9. Natural Log $y = \ln x$



Domain:

Range:

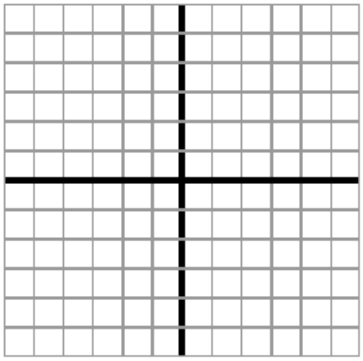
10. Rational $y = \frac{1}{x}$



Domain:

Range:

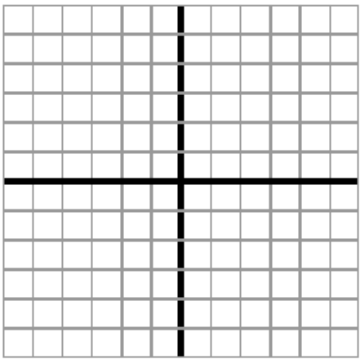
11. Inverse Squared $y = \frac{1}{x^2}$



Domain:

Range:

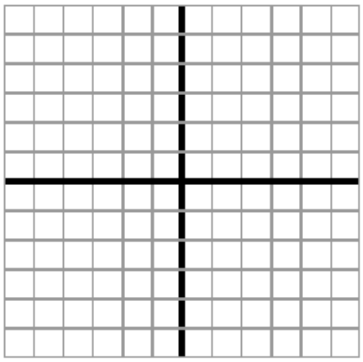
12. Greatest Integer $y = \lfloor x \rfloor$



Domain:

Range:

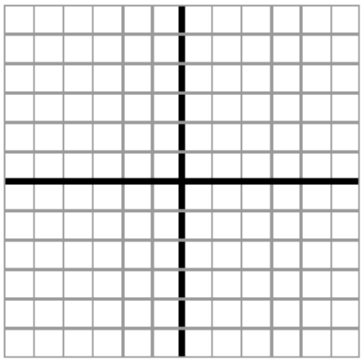
13. Sine $y = \sin x$



Domain:

Range:

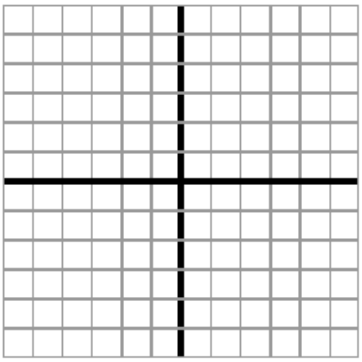
14. Cosine $y = \cos x$



Domain:

Range:

15. Tangent $y = \tan x$



Domain:

Range:

Topic 6: Function Transformations

Describe in words and algebraically what the following transformations would do to a graph of $f(x)$.

1. $f(x)-4$

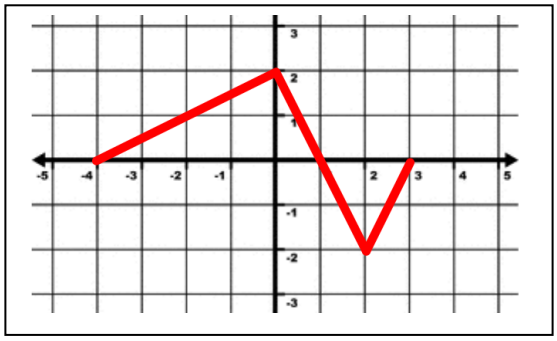
2. $f(x-3)$

3. $-f(6x)$

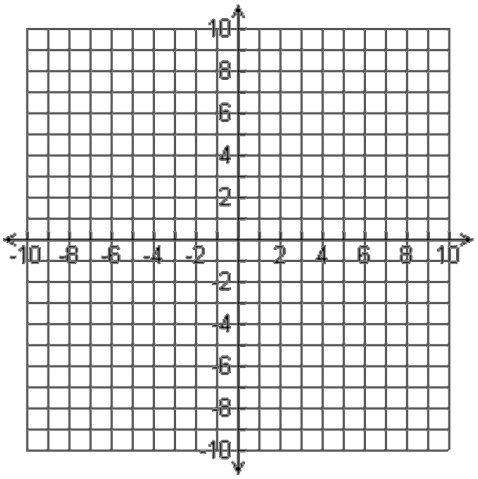
4. $5f(x)+3$

5. $|f(x)|$

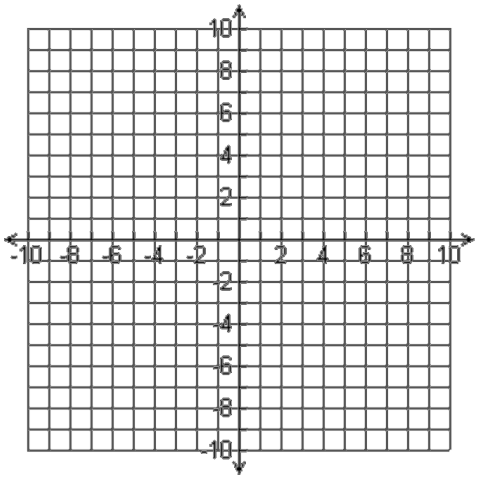
To the right is a graph of $y=f(x)$. Sketch the following transformation graphs based on this and describe in words the transformations taking place.



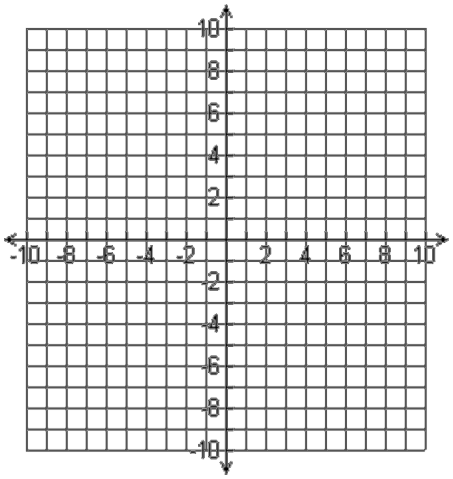
6. $y=2f(x)$



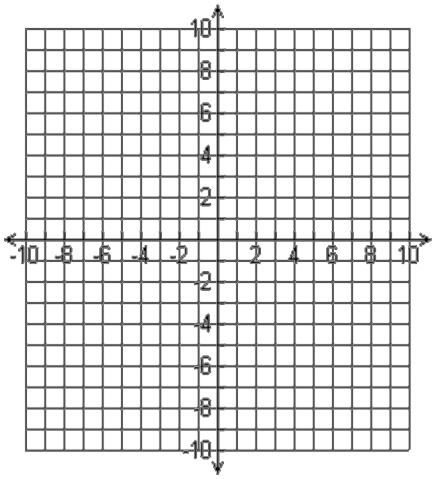
7. $y=f(-x)$



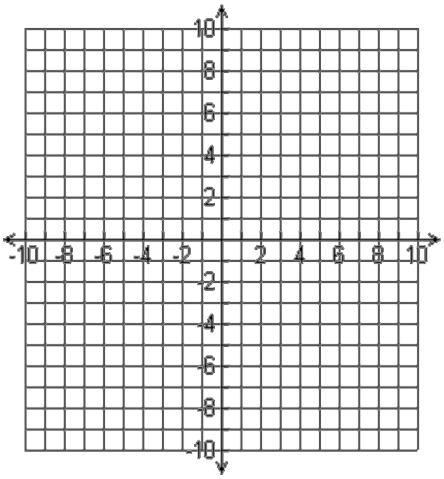
8. $y=f(x+1)$



9. $y=f(2x)+6$



10. $y=-f(x-5)$



Topic 7: Composition of Functions

If $f(x)=x^2$, $g(x)=2x-1$, and $h(x)=2^x$, find the following…

1. $f(g(2))$

2. $g(h(3))$

3. $f(h(-1))$

4. $h(f(-2))$

5. $g\left(f\left(h\left(\frac{1}{2}\right)\right)\right)$

6. $f(h(g(-1)))$

7. $g(f(x))$

8. $g(g(x))$

9. $f(h(x))$

10. $f(g(x))$

Topic 8: Unit Circle

1. $\sin\frac{5\pi}{6} =$
2. $\cos\frac{5\pi}{3} =$
3. $\sin\frac{5\pi}{4} =$
4. $\cos\frac{3\pi}{4} =$
5. $\sin\frac{2\pi}{3} =$
6. $\cos\frac{\pi}{2} =$
7. $\cos\frac{4\pi}{3} =$
8. $\sin\frac{4\pi}{3} =$
9. $\sin\frac{7\pi}{4} =$
10. $\cos\frac{7\pi}{6} =$
11. $\sin\frac{3\pi}{2} =$
12. $\cos\pi =$
13. $\tan\frac{5\pi}{4} =$
14. $\tan\frac{\pi}{3} =$
15. $\tan\frac{7\pi}{6} =$
16. $\tan\pi =$
17. $\tan\frac{2\pi}{3} =$
18. $\tan\frac{\pi}{2} =$
19. $\tan\frac{7\pi}{4} =$
20. $\tan\frac{11\pi}{6} =$
21. $\cos\frac{\pi}{4} =$
22. $\csc\frac{\pi}{6} =$
23. $\cot\frac{2\pi}{3} =$
24. $\sec\frac{4\pi}{3} =$
25. $\csc\pi =$
26. $\cot\frac{7\pi}{6} =$
27. $\sec\frac{\pi}{2} =$
28. $\csc\frac{3\pi}{2} =$
29. $\cot\frac{3\pi}{2} =$
30. $\sec\frac{5\pi}{6} =$
31. $\csc\frac{3\pi}{4} =$
32. $\cot\pi =$
33. $\sec\frac{\pi}{6} =$

Topic 9: Inverse Trig

Answer each within the "shaded area" or allowed domain. There should be only one answer per question.

1. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$
2. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) =$
3. $\cos^{-1}\left(-\frac{1}{2}\right) =$
4. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$
5. $\tan^{-1}(1) =$
6. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$
7. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$
8. $\sin^{-1}(-1) =$
9. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) =$
10. $\tan^{-1}(0) =$
11. $\cos^{-1}(0) =$
12. $\sin^{-1}\left(-\frac{1}{2}\right) =$
13. $\tan^{-1}(-1) =$
14. $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) =$
15. $\sin^{-1}(0) =$
16. $\cos^{-1}(-1) =$
17. $\tan^{-1}\left(-\sqrt{3}\right) =$
18. $\cos^{-1}\left(\frac{1}{2}\right) =$
19. $\sec^{-1}(-2) =$
20. $\csc^{-1}\left(\frac{2}{\sqrt{3}}\right) =$
21. $\cot^{-1}\left(-\sqrt{3}\right) =$
22. $\csc^{-1}(2) =$
23. $\cot^{-1}(undef) =$
24. $\sec^{-1}\left(-\frac{2}{\sqrt{2}}\right) =$

Topic 10: Solving Trigonometric Equations

Solve each equation on the interval $[0, 2\pi)$ with radian answers.

1. $\sin x = \frac{1}{2}$

2. $2\cos x + \sqrt{3} = 0$

3. $3\tan^2 x + 2 = 11$

4. $4\sin^2 x = 1$

5. $\cos^2 x = \cos x$

6. $2\sin^2 x + \sin x = 1$

7. $\cos^2 x + 2\cos x = 3$

8. $2\sin x \cos x = -\sin x$

9. $\sin^2 x - \cos^2 x = 0$

10. $2\tan x + 1 - 3\tan^2 x = -4\tan^2 x$

ANSWER KEY

Topic 1

1. $\frac{3x^2}{y^3}$
2. $3x^4y^5$
3. $\frac{12}{n^2}$
4. $8x^5$
5. 2
6. $\frac{1}{ab^2}$
7. $4a^{\frac{15}{4}}b^{\frac{31}{12}}$ *or* $4\sqrt[4]{a^{15}}\sqrt[12]{b^{31}}$
8. $\frac{x^{\frac{1}{2}}}{y^{\frac{7}{3}}}$ *or* $\frac{\sqrt{x}}{\sqrt[3]{y^7}}$
9. $\frac{a^4}{4b^{18}}$
10. $\frac{x^2}{9}$
11. $\frac{1}{16b^2}$
12. $\frac{v}{8u^{\frac{2}{3}}}$ *or* $\frac{v}{8\sqrt[3]{u^2}}$

Topic 2

1. $(x+7)(x+8)$
2. $(x-5)(x+2)$
3. $(x+9)(x-8)$
4. $(x-6)(x-5)$
5. $(x+3)(x-2)$
6. $(x-9)(x-4)$
7. $(x+20)(x-5)$
8. $(x-8)(x+6)$
9. $-(x+9)(x-5)$
10. $-x(x-10)(x+8)$
11. $4x^2(x+6)(x-5)$
12. $4x(x-4)^2$
13. $(2x+1)(x-2)$
14. $(3x-2)(x-3)$
15. $(5x+3)(x+1)$
16. $(4x+5)(x-1)$
17. $(5x+2)(2x+3)$
18. $(3x+4)(2x+3)$
19. $(5x+2)(x-2)$
20. $(2x+5)(2x+1)$

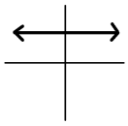
Topic 3

1. $(x+2)(x^2-2x+4)$
2. $(x-4)(x^2+4x+16)$
3. $(3x-5y)(9x^2+15xy+25y^2)$
4. $(x+4)(x-4)(x^2+5)$
5. $(x-3)(x+3)^2$
6. $(7a-1)(5b-8)$
7. $(3x^2+5)(2x^2+1)$
8. $(x+3)(x-3)(x+2)(x-2)$
9. $(11x+20)(11x-20)$
10. $(15+4x)(15-4x)$

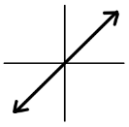
Topic 4

1. $x=0,\ x=\frac{3}{7}$
2. $x=2,\ x=-10$
3. $x=-2,\ x=\frac{7}{3}$
4. $x=-1,\ x=-2$
5. $x=-3-\sqrt{5}$ $x=-3+\sqrt{5}$
6. $x=-3,\ x=2$
7. $x=\frac{2}{3},\ x=\frac{3}{2}$
8. $x=\frac{1}{3},\ x=-\frac{1}{2}$
9. $x=-1,\ x=1$ $x=-2\sqrt{2},\ x=2\sqrt{2}$
10. $x=1,\ x=81$

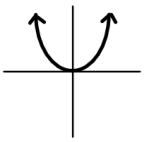
Topic 5

1. 

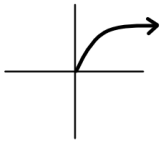
Domain: $\mathbb{R}$, Range: number

2. 

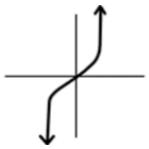
Domain: $\mathbb{R}$, Range: $\mathbb{R}$

3. 

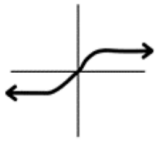
Domain: $\mathbb{R}$, Range: $[0,\infty)$

4. 

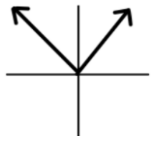
Domain: $[0,\infty)$, Range: $[0,\infty)$

5. 

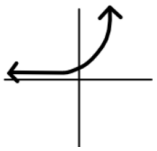
Domain: $\mathbb{R}$, Range: $\mathbb{R}$

6. 

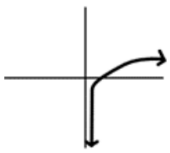
Domain: $\mathbb{R}$, Range: $\mathbb{R}$

7. 

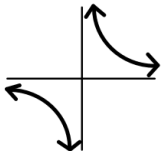
Domain: $\mathbb{R}$, Range: $[0,\infty)$

8. 

Domain: $\mathbb{R}$, Range: $(0,\infty)$

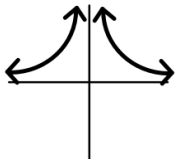
9. 

Domain: $(0,\infty)$, Range: $\mathbb{R}$

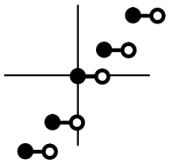
10. 

Domain: $(-\infty,0)(0,\infty)$

Range: $(-\infty,0)(0,\infty)$

11. 

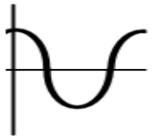
Domain: $(-\infty,0)(0,\infty)$, Range: $(0,\infty)$

12. 

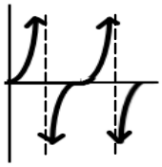
Domain: $\mathbb{R}$, Range: $\mathbb{Z}$

13. 

Domain: $\mathbb{R}$, Range: $[-1,1]$

14. 

Domain: $\mathbb{R}$, Range: $[-1,1]$

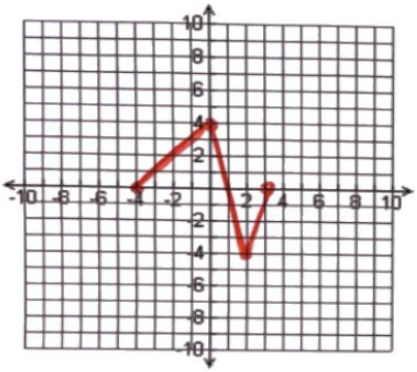
15. 

Domain: $\mathbb{R}$, $x \neq \frac{\pi}{2} + \pi k$, Range: $\mathbb{R}$

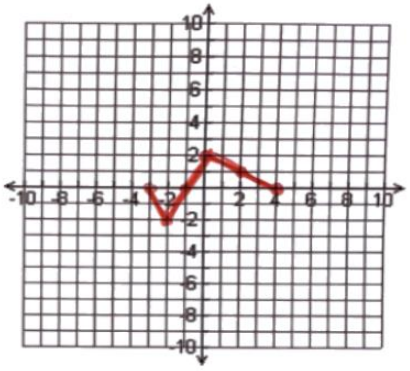
Topic 6

1. vertical shift down 4 (y-coordinate minus 4)
2. horizontal shift right 3 (x-coordinate plus 3)
3. vertical reflection over the x-axis (y-coordinate times -1) & horizontal compression by 1/6 (x-coordinate times 1/6)
4. vertical stretch by 5 (y-coordinate times 5) & vertical shift up 3 (y-coordinate plus 3 AFTER times 5)
5. parts of the graph below the x-axis reflect up while parts of the graph above the x-axis are left alone (negative y-coordinate times -1, positive y-coordinate left alone)
6. vertical stretch by 2
7. horizontal reflection over the y-axis
8. horizontal shift left 1
9. horizontal compression by 1/2 & vertical shift up 6
10. vertical reflection over the x-axis & horizontal shift right 5

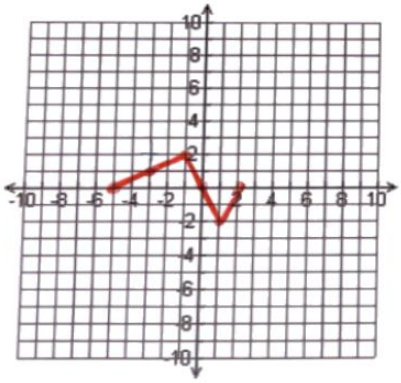
6. $y = 2f(x)$



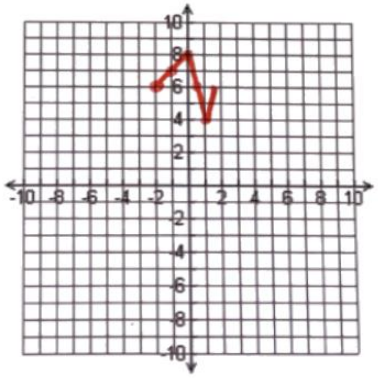
7. $y = f(-x)$



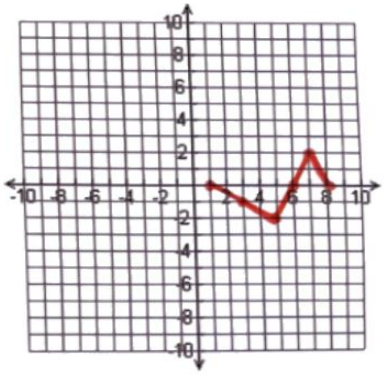
8. $y = f(x+1)$



9. $y = f(2x)+6$



10. $y = -f(x-5)$



Topic 7

1. 9
2. 15
3. $\frac{1}{4}$
4. 16
5. 3
6. $\frac{1}{64}$
7. $2x^2 - 1$
8. $4x - 3$
9. 4^x
10. $4x^2 - 4x + 1$

Topic 8

1. $\frac{1}{2}$
2. $\frac{1}{2}$
3. $-\frac{\sqrt{2}}{2}$
4. $-\frac{\sqrt{2}}{2}$
5. $\frac{\sqrt{3}}{2}$
6. 0
7. $-\frac{1}{2}$
8. $-\frac{\sqrt{3}}{2}$
9. $-\frac{\sqrt{2}}{2}$
10. $-\frac{\sqrt{3}}{2}$
11. -1
12. -1
13. 1
14. $\sqrt{3}$
15. $\frac{1}{\sqrt{3}}$
16. 0
17. $-\sqrt{3}$
18. undefined
19. -1
20. $-\frac{1}{\sqrt{3}}$
21. $\frac{\sqrt{2}}{2}$
22. 2
23. $-\frac{1}{\sqrt{3}}$
24. -2
25. undefined
26. $\sqrt{3}$
27. undefined
28. -1
29. 0
30. $-\frac{2}{\sqrt{3}}$
31. $\frac{2}{\sqrt{2}}$
32. undefined
33. $\frac{2}{\sqrt{3}}$

Topic 9

1. $\frac{\pi}{3}$
2. $\frac{\pi}{6}$
3. $\frac{2\pi}{3}$
4. $-\frac{\pi}{4}$
5. $\frac{\pi}{4}$
6. $\frac{5\pi}{6}$
7. $-\frac{\pi}{3}$
8. $-\frac{\pi}{2}$
9. $\frac{\pi}{4}$
10. 0
11. $\frac{\pi}{2}$
12. $-\frac{\pi}{6}$
13. $-\frac{\pi}{4}$
14. $\frac{\pi}{4}$
15. 0
16. π
17. $-\frac{\pi}{3}$
18. $\frac{\pi}{3}$
19. $\frac{2\pi}{3}$
20. $\frac{\pi}{3}$
21. $-\frac{\pi}{6}$
22. $\frac{\pi}{6}$
23. 0
24. $\frac{3\pi}{4}$

Topic 10

1. $x=\frac{\pi}{6}, \frac{5\pi}{6}$
2. $x=\frac{5\pi}{6}, \frac{7\pi}{6}$
3. $x=\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
4. $x=\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
5. $x=\frac{\pi}{2}, \frac{3\pi}{2}$ and $x=0$
6. $x=\frac{\pi}{6}, \frac{5\pi}{6}$ and $x=\frac{3\pi}{2}$
7. $x=0$ (other factor yields N/A)
8. $x=0, \pi$ and $x=\frac{2\pi}{3}, \frac{4\pi}{3}$
9. $x=\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
10. $x=\frac{3\pi}{4}, \frac{7\pi}{4}$